Chapter 6 - Day 5

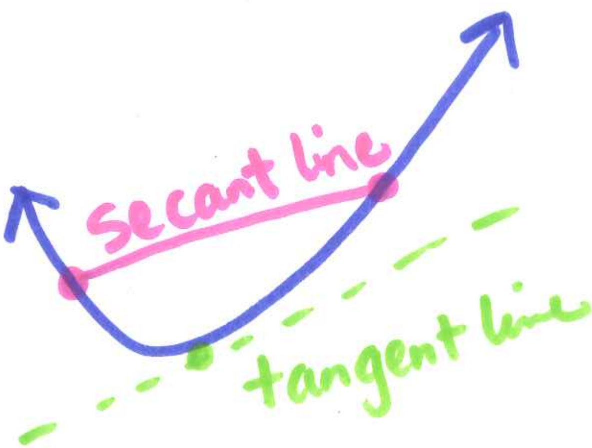
The first derivative gave us info on increasing/decreasing.

The 2^{nd} derivative gives us info on concavity.

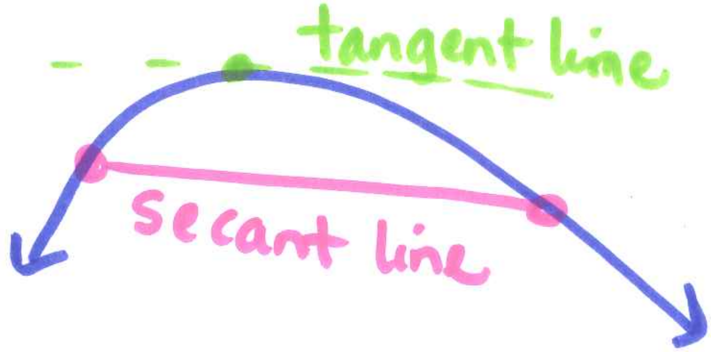
We say $y=f(x)$ is <u>concave up</u> on interval I if for points $a<b$ in I, the secant line through $(a, f(a))$ and $(b, f(b))$ lies above the graph of $y=f(x)$ for $a<x<b$.

$y = f(x)$ is <u>concave down</u> on interval I if for $a<b$ in I, the secant line through $(a, f(a))$ and $(b, f(b))$ lies below the graph of $y=f(x)$ for $a<x<b$.

Also, note the tangent lines:



concave up



concave down

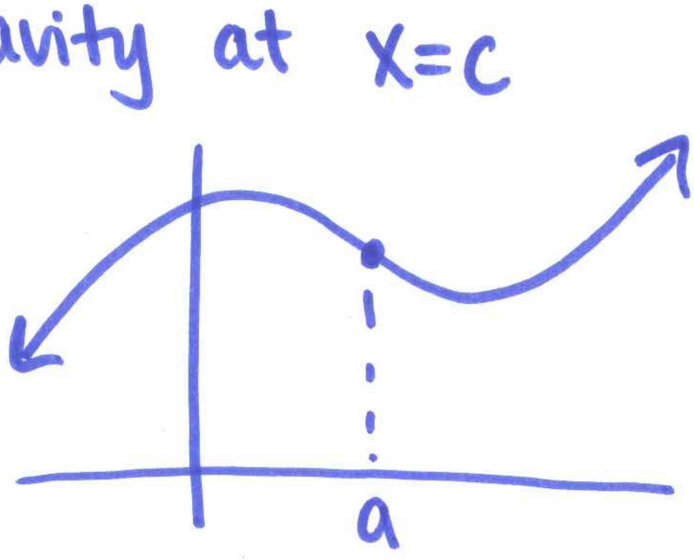
Second Derivative Test for Concavity:

- $y=f(x)$ is concave up on $[a,b]$ if and only if $f''(x) \geq 0$ for all $x \in [a,b]$

- $y=f(x)$ is concave down on $[a,b]$ if and only if $f''(x) \leq 0$ for all $x \in [a,b]$

A point $(c, f(c))$ on the graph is called a <u>point of inflection</u> if the graph of $y=f(x)$ changes concavity at $x=c$

<u>Ex:</u>



f concave down $(-\infty, a)$

concave up (a, ∞)

a is an inflection Point

Ex: Sketch a graph of $f(x) = x^4$

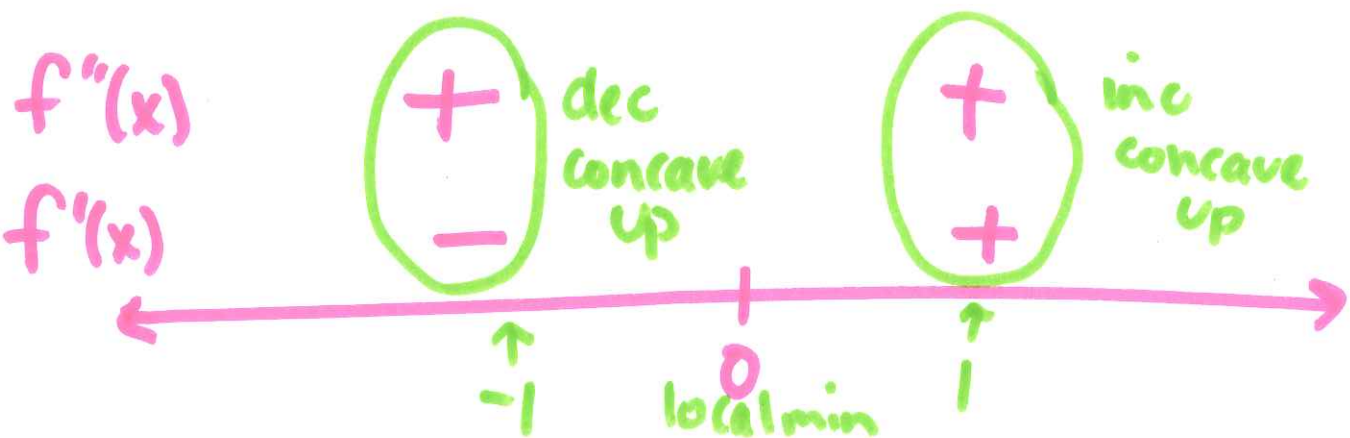
$f(x)$ defined everywhere ✓

$f'(x) = 4x^3$ defined everywhere ✓

$f'(x) = 0$ when $x = 0$

$f''(x) = 12x^2$ defined everywhere ✓

$f''(x) = 0$ when $x = 0$



$f'(-1) = 4(-1)^3 = -4$ "−"

$f'(1) = 4(1)^3 = 4$ "+"

$f''(-1) = 12(-1)^2 = 12$ "+"

$f''(1) = 12(1)^2 = 12$ "+"

$f(x)$ increasing on $(0, \infty)$

decreasing on $(-\infty, 0)$

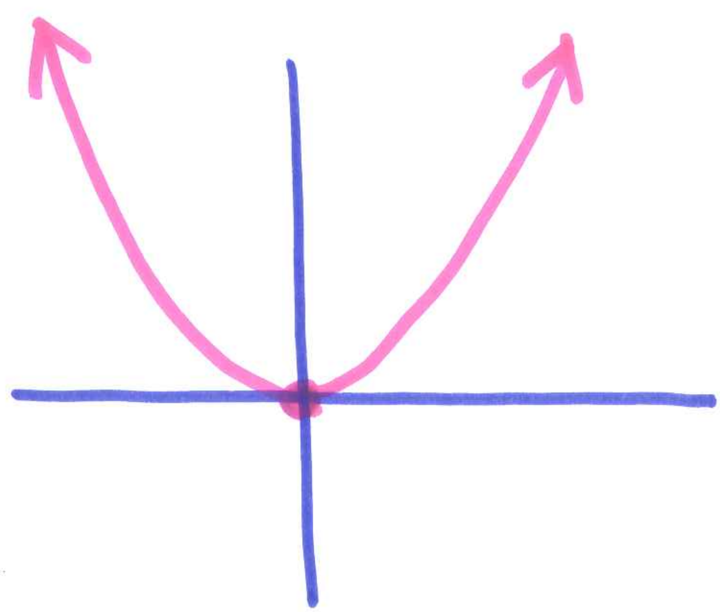
Concave up $(-\infty, \infty)$

Concave down nowhere

0 is a local min but not an inflection point.

It is helpful to know $f(0) = 0^4 = 0$

so the graph contains the point $(0,0)$



Ex: Sketch the graph of $f(x) = xe^{-x}$

$$f'(x) = (1)(e^{-x}) + (x)(e^{-x})(-1)$$ *product rule

$$= e^{-x} - xe^{-x}$$

$$= e^{-x}(1-x)$$

$f'(x) = 0$ when $x = 1$

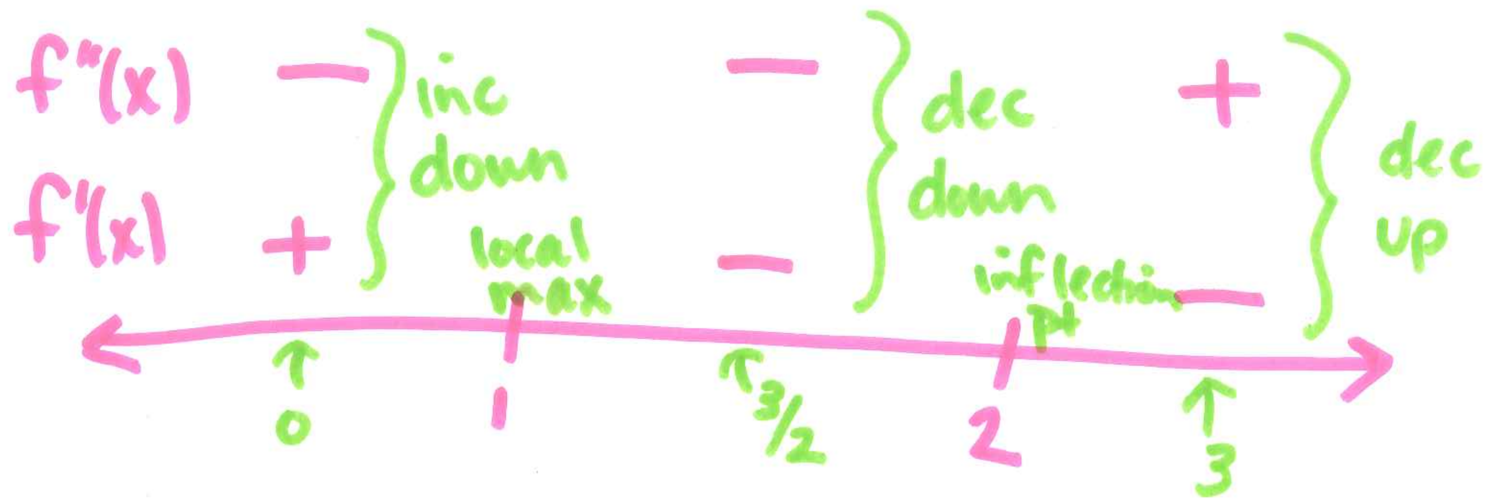
$$f''(x) = (e^{-x})\underline{(-1)(1-x)} + (e^{-x})(-1)$$

$$= e^{-x}(x-1) - e^{-x}$$

$$= e^{-x}((x-1)-1)$$

$$= e^{-x}(x-2)$$

$f''(x) = 0$ when $x = 2$



$f'(0) = e^0(1-0) = 1 \cdot 1 = 1$ "+"

$f'(\frac{3}{2}) = (+)(-) = -$

$f'(3) = (+)(-) = -$

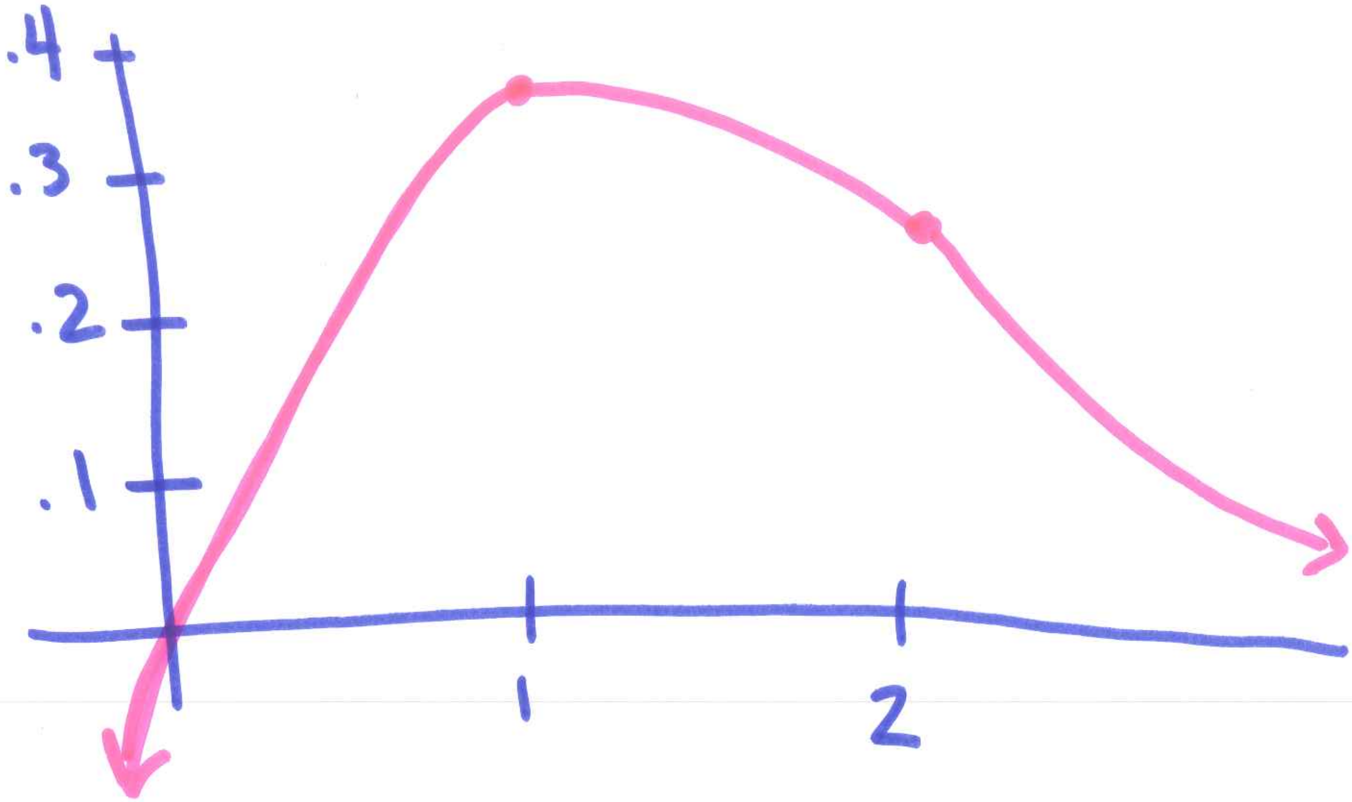
$f''(0) = (+)(-) = -$

$f''(\frac{3}{2}) = (+)(-) = -$

$f''(3) = (+)(+) = +$

$f(1) = 1e^{-1} = \frac{1}{e} \approx .368$

$f(2) = 2e^{-2} = \frac{2}{e^2} \approx .271$


.4
.3
.2
.1
1
2